Using 21cm Emission to Map the Spiral Structure of the Milky Way Galaxy

Joan C. Smith*
MIT Department of Physics
(Dated: April 4, 2011)

The partial spiral arm structure of the Milky Way Galaxy can be mapped with 21cm wavelength radiation produced by the hyperfine transition in hydrogen. Using a model for velocity as a function of radius from the center of the galaxy the location of a spiral arm can be determined. In this experiment a single spiral arm was found between 60° and 155° galactic latitude, at 8kpc from the galactic center. The spiral arm extends at least through +10° through -10°. This confirms the Oort (1958) spiral arm found between 5-10kpc from the galactic center found between 65°-135° galactic longitude[1].

1. INTRODUCTION

The hyperfine splitting of hydrogen produces 21cm wavelength radiation that can be used to study features of the galaxy that are visibly obscured by gas and dust. 21cm wavelength (1420.4MHz frequency) radiation penetrates the gas and dust prevalent in the galaxy.

We claim that the galaxy in the region of galactic longitude 60° to 155° and galactic latitude -10° to 10° has a spiral arm structure that can be mapped using 21cm radio astronomy.

This spiral arm partially confirms the spiral arm in Oort, 1958[1]. In identifying a spiral arm, a portion of the rotation curve for the galaxy was also plotted and confirmed to match the accepted data.

1.1. Hyperfine Splitting of Hydrogen

The hyperfine splitting of hydrogen arises from the magnetic field produced by the proton acting on the electron. The following description of hyperfine splitting follows Griffith's derivation[2]. Classically, the magnetic field produced by a dipole is $B = \frac{\mu_0}{4\pi r^3}[3(\vec{\mu}\cdot\hat{r})\hat{r} - \mu] + \frac{2\mu_0}{3}\mu\delta^3(\vec{r})$, where μ_0 is the vacuum permeability, μ is the magnetic moment of the dipole, $\vec{r}$ is the vector from the dipole to the place the field is being measured, r is that vector's magnitude and $\hat{r} = \frac{\vec{r}}{r}$ is the unit vector parallel to $\vec{r}$. In the case of hydrogen, the single electron and proton are interacting. To model hyperfine splitting the magnetic moment of the electron and proton are needed and the magnetic field produce by a dipole (the proton). μ_e, the electron's magnetic moment is

$$\mu_e = -\frac{e}{m_e}\hat{\vec{S}}_e \tag{1}$$

where e is the charge of the electron, m_e is the mass of the electron, and $\hat{\vec{S}}_e$ is the spin of the electron. Similarly, the magnetic moment of the proton is

$$\mu_p = \frac{g_p e}{2m_p}\hat{\vec{S}}_p \tag{2}$$

$g_p = 5.59$ is the g-factor that accounts for the composite nature of protons[2]. The result is an interaction between two spins: the spin of the proton and the spin of the electron. The Hamiltonian for hyperfine splitting can be written as a perturbation of the hydrogen Hamiltonian

$$\delta H_{hf} = \frac{\mu_0 g_p e^2}{8\pi m_p m_e}\frac{[3(\hat{\vec{S}}_p\cdot\hat{r})(\hat{\vec{S}}_e\cdot\hat{r}) - (\hat{\vec{S}}_p\cdot\hat{\vec{S}}_e)]}{r^3} \tag{3}$$

$$+\frac{\mu_0 g_p e^2}{3m_p m_e}\left(\hat{\vec{S}}_p\cdot\hat{\vec{S}}_e\delta^3(\vec{r})\right) \tag{4}$$

Using perturbation theory the first order energy shift is calculated to be the expectation value of Eq. 3 with the wave functions of the unpreturbed Hamiltonian. When the wave function is spherically symmetric, (as in the ground state) the first term of the expectation value vanishes, leaving

$$E^1_{hf} = \frac{\mu_0 g_p e^2}{3\pi m_p m_e a^3}\langle\hat{\vec{S}}_p\cdot\hat{\vec{S}}_e\rangle \tag{5}$$

Here $a = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2}$, ϵ_0 is the permittivity of free space, and μ_0 is the permeability of free space. The spin-spin coupling is in the $\langle\hat{\vec{S}}_p\cdot\hat{\vec{S}}_e\rangle$ term. Using total spin states, or the $|nlm_js_j\rangle$ basis, simplifies the problem. The perturbation expectation value can be written in terms of eigen-operators in this basis: $\hat{\vec{S}}_p\cdot\hat{\vec{S}}_e = \frac{1}{2}(S^2 - S_e^2 - S_p^2)$. For the ground state $\hat{\vec{S}}_p = \hat{\vec{S}}_e = \frac{3}{4}\hbar^2$, and S^2 can take two values. $S^2 = 0$ when both the proton and electron spin are in the singlet state, and $S^2 = 2\hbar^2$ in the triplet state. Since S^2 takes a different value for the singlet and triplet states, the spin degeneracy is broken.

$$E^1_{hf} = \frac{4g_p\hbar^4}{3m_p m_e^2 c^2 a^4}\begin{cases} +\frac{1}{4} & \text{triplet} \\ -\frac{3}{4} & \text{singlet}\end{cases}$$

Where g_p, m_e, m_p are as above, c is the speed of light, a is the fine structure constant also defined above.

When a hydrogen atom transitions from the higher energy hyperfine state (the triplet) into the lower energy

*Electronic address: joans@mit.edu

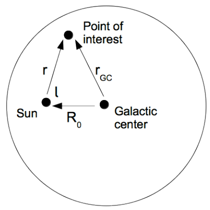


FIG. 1: The geometry for relating galactic radius and distance from the sun.

state it releases a photon with energy $\Delta E = \frac{4g_p\hbar^4}{3m_p m_e^2 c^2 a^4} = 5.88 \times 10^{-6}$eV, or $\nu = \frac{\Delta E}{h} = 1420$MHz= 21cm.

The hyperfine transition that produces 21cm radiation is highly forbidden. In a single hydrogen atom the transition will occur approximately once per 10^7 years. Due to the vast number of hydrogen atoms in the galaxy this transition occurs sufficiently often that 21cm radiation can be used to study galactic properties.

1.2. Galactic Models

21cm radiation is used in this experiment to map the structure of the galaxy. The velocity of hydrogen atoms with respect to the earth can be calculated by observing the change in frequency due to Doppler shift. Calculating the velocity of hydrogen clouds allows the creation of a model relating velocity to radius. Once this model is prepared it can be used to show the spiral arms of the galaxy.

The change in frequency due to Doppler effects is $\Delta f = f\frac{v}{c}$. Where f is the observed frequency, v is the velocity of the object and c is the speed of light.

The Milky Way galaxy is a flat disk with a central bulge containing a supermassive black hole.

The data collected in this experiment are frequency spectra taken at a variety of galactic latitude and longitudes, and show Doppler shifts based on the velocity of hydrogen clouds. Throughout this paper the galactic coordinate system will be used. Galactic longitudes are angles denoted l measured from the sun to a galactic point. Galactic latitudes, called b give the angular distance between a point and the galactic plane.

In this experiment, we will calculate the rotational velocity for points about the center of the galaxy. The model from Contopolous and Strömgren, from the NASA Institute for Space Studies[3], is used to relate velocity of galactic points to radius from the galactic center. This model assumes that the galaxy rotates as a uniform disk, so that each galactic radius has a single constant velocity. The functional form of the model is

$$v(r) = 0.0861r^3 - 4.0448r^2 + 50.56r + 67.76 \qquad (6)$$

This experimental model will allow the calculation of radii for data points for which the velocity is known.

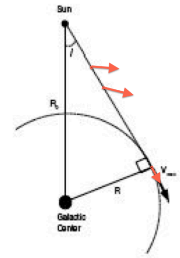


FIG. 2: Schematic for calculating velocities from line of sight spectra. Red arrows indicate individual hydrogen clouds that have calculable velocities. Red arrow forming the right triangle allows the calculation of maximum recessional velocity. The other two arrows are hydrogen clouds whose velocities are calculated by the second method.

1.3. Computing Velocities and Radii

Frequency spectra are collected in this experiment and must be converted to velocity spectra to apply the Contopolous and Strömgren model. Velocities can be calculated from frequency using Doppler shift. For any frequency measured in this experiment, the corresponding velocity can be calculated

$$v(f,l) = \frac{c(f_0^2 - f^2)}{f_0^2 + f^2} - v_{lsr} + v_s \sin l + v_e \qquad (7)$$

Where f is the observed frequency, l is the galactic longitude, $f_0 = 1420.4$MHz, $v_s = 220 \pm 20$km/s is the velocity of the velocity of the sun about the galactic center, and $v_e = 29.8 \pm .5$km/s is the velocity of the earth about the sun. v_{lsr} is the velocity of the local standard of rest and measures the difference in the velocity of the sun from a perfectly circular orbit about the galactic center.

As shown in Fig. 2, radii for points of maximum recessional velocity can be calculated. Other points must use the geometry shown in Fig. 1.

2. EXPERIMENTAL SETUP

This experiment uses a Haystack Small Radio Telescope mounted on Building 26 on the Massachusetts Institute of Technology campus. The telescope is controlled by the Small Radio Telescope software system that provides interfacing, data acquisition and basic analysis.

2.1. Telescope Overview

The Haystack SRT has a diameter of 2.3m and a half power beam width of 7°. As shown in the block diagram in Fig. 3, this signal is reflected and focused to the antenna feed horn. The signal passes through a band-pass filter in order to select a band of the approximate frequency at which data is being taken, a cavity filter, and

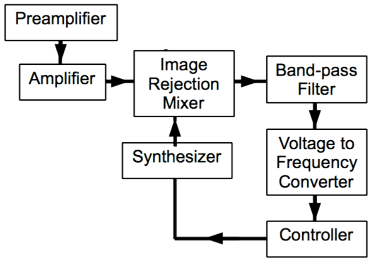


FIG. 3: The signal chain for filtering of the 21cm signal.

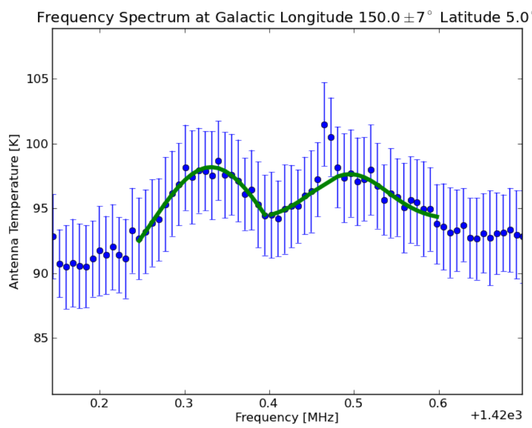
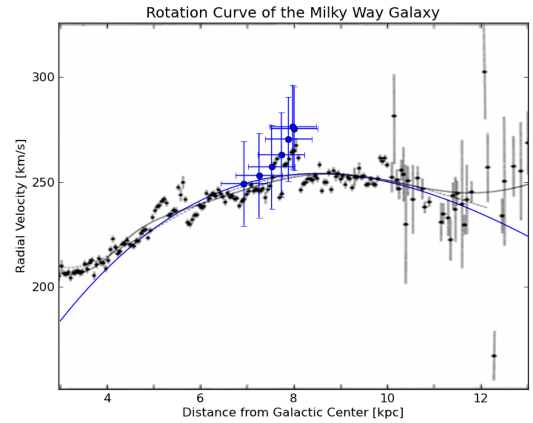


FIG. 4: An example of a single frequency spectrum. Data taken at $l = 150°$, $b = 5°$. Error bars are standard deviations of intensity for each 5s data collection over 10 minute integration time.

a mixer. The cavity filter was added to the signal chain for this experiment. The local oscillator frequency used by the mixer can be modified by the software program, allowing an experimenter to look at varying wavelengths close to 21cm. After pass- ing through the mixer, the signal is sent through a serial cable to a computer running software written in Java.

2.2. Filtering and Electronics

The cavity filter was added and tuned to minimize interference from nearby cellular phone repeaters. Before taking data the cavity filter was manually tuned using a signal generator operating at 1420.4MHz. The filter was tuned to allow only the signal from the generator to be picked up by the other filtering electronics. The result is exceptionally clean spectra from a telescope located in a densely populated area with heavy radio noise.



cmcm

FIG. 5: Overlay of the Clemens[4] data and the experimental data. The peak around 8kpc in our experimental data matches the peak at 8kpc in the accepted data.

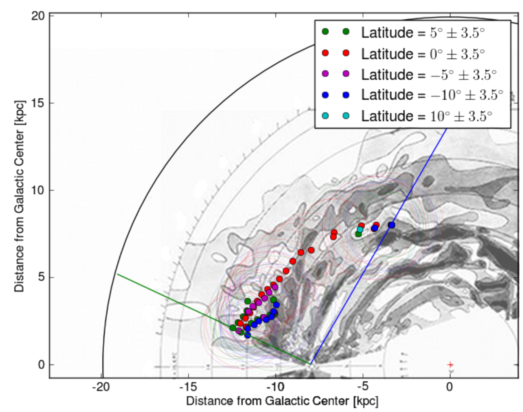


FIG. 6: The spiral arm data collected in this experiment overlaid on the original Oort 21cm map of galactic hydrogen [1]. The data points colors correspond to the galactic latitude of the points. The error bars on our experimental data are marked as circles that define the possible error. Radii of the error circles are determined by propagating error on the mean of the fit Gaussians and the uncertainty in measurement of the constant parameters.

3. DATA AND ANALYSIS

Data collected in this experiment are frequency spectra centered at 1420.4MHz. Each spectra has 168 frequency bins separated by 0.0078125MHz. Data was taken between 60° and 155° galactic longitude in 5° increments at each -10°, 5°, 0°, 5°, 10° galactic latitude in three 6-10 hour sessions. Integration times for each spectra were 10 minutes. A sample spectra is shown in Fig. 4. Each spectra was examined and the obvious peaks were fit to Gaussians with the functional form

$$I(f) = d + ae^{\frac{-(f-f_0)^2}{2\sigma^2}} \tag{8}$$

Here I is the intensity of the peaks (y-axis) and f is the frequency on the x-axis. The Gaussian fits allowed the calculation of two kinds of velocities. The peak with the largest peak frequency (smallest peak velocity) for data points between 60° and 90° in the galactic plane was used to calculate the maximum recessional velocity an equivalent rectangle method defined in Tuve and Lundsager [5]. Those peaks and all the others were used to calculate the spiral arm structure of the galaxy.

3.1. Rectangle-Method for Maximum Velocity

Calculating the maximum recessional velocity allows the confirmation of the Contopolous and Strömgren model for velocity as a function of radius from the center of the galaxy. The geometry for this calculation is shown in Fig. 2. The distance between the sun and and the galactic center is known, as is the angle from the sun between the galactic center and the relevant point. The maximum recessional velocity occurs on a line that is tangent to the circle centered at the galactic center because the velocity of the hydrogen cloud can be taken to be entirely in the direction parallel to the line of sight. Straightforward trigonometry then allows the calculation of the distance between the relevant point and the galactic center.

The calculation of the maximum recessional velocity follows Tove and Lundsager [5]. To find the velocity a numerical integral is taken over the data points from the peak of the fitted Gaussian to the minimum of the frequency spectra. The result of this integral is divided by the height of the Gaussian. This defines the width of a rectangle whose height is the height of the fitted Gaussian and whose area is the area under the data points. The width defines the maximum recessional velocity.

3.2. Spiral Arm Structure

Once the maximum recessional velocity has been calculated for each relevant data point and the rotation curve is built, radii for points whose velocities are not entirely along the line of sight of the galaxy must be calculated. First, the radius of the point from the galactic center is calculated using the Contopolous and Strömgren model. The velocities from the spectra are used as v in Eq. 6, and the roots of the equation are found. Once the radius from the galactic center is found (r_{GC}) a transformation is applied to convert r_{GC} to r, the radius from the sun. This is necessary because the galactic coordinate system defines angles about the sun. The geometry for the transformation is shown in Fig. 1 and the relationship is defined using the law of sines.

$$r = \frac{r_{GC}}{\sin l} \arcsin\left(\pi - l - \sin\frac{R_0 \sin l}{r_{GC}}\right) \tag{9}$$

Here $R_0 = 8.0 \pm .5$kpc is the distance from the sun to the galactic center.

In Fig. 6 the partial spiral arm structure of the galaxy is plotted with $r\cos l$ on the x-axis and $r \sin l$ on the y-axis. This data is overlaid on data from Oort, 1958[1]. The partial spiral arm is clearly visible in our data and matches the spiral arm in the in the Oort data.

3.3. Error Analysis

The largest source of systematic error in measurements is the uncertainties in known quantities such as v_s and R_0. Random error is accounted for by taking the standard deviation of intensity over ten minutes of integration time for each galactic latitude and longitude position. The beam width of the telescope also provides imprecise measurements. Errors were propagated through calculation according to Bevington [6]

4. CONCLUSIONS

The partial spiral arm structure of the Milky Way Galaxy can be mapped with 21cm wavelength radiation produced by the hyperfine transition in hydrogen. Using a model for velocity as a function of radius from the center of the galaxy, the location of a spiral arm can be determined. In this experiment the model from Contopolous and Strömgren is used and a single spiral arm was found between 60° and 155° galactic latitude, at 8kpc from the galactic center. The spiral arm extends at least through +10° through -10°. This confirms the Oort (1958) spiral arm found between 5-10kpc from the galactic center found between 65°-135° galactic longitude[1].

[1] J. H. Oort, F. J. Kerr, and G. Westerhout, MNRAS (1958).
[2] D. J. Griffiths, *Introduction to Quantum Mechanics* (Prentice Hall, 2005), 2nd ed.
[3] G. Contopolous and B. Strömgren, NASA Institute for Space Studies (1965).
[4] D. P. Clemens, Astrophys. J. (1985).
[5] M. A. Tuve and S. Lundsager, Carnegie Institute of Washington Publication (1973).
[6] P. Bevington and D. Robinson, *Data Reduction and Error Analysis for the Physical Sciences* (McGraw-Hill, 2003).