21cm Astrophysics

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In this experiment we use a radio telescope collecting data at 21cm in order to measure the brightness temperature of the sun, analyze the diffraction pattern of the telescope and measure the rotation curve of the Milky Way galaxy. The calculated brightness temperature of the sun at 21cm is $T_b = (4.5 \pm 0.3)10^4 K$ the diffraction pattern appears strongly as expected. The circular aperture of the telescope is measured to be $255 \pm 3.2cm$. The galactic rotation curve disagrees with the Keplerian prediction for the rotation curve of the galaxy,sugesting the existence dark matter, and implying the spiral arm structure of the galaxy.

1. INTRODUCTION

In this experiment the parabolic dish antenna was used to measure the 21cm line of interstellar hydrogen emission. Interstellar hydrogen emission can be used to measure the brightness temperature of the sun at 21cm, calculate some physical properties of the telescope and determine the rotation curve of the galaxy. 21cm wavelength radiation is due to hyperfine splitting of the hydrogen atom, and hydrogen is found in abundance in the galaxy. Therefore 21cm radiation is useful for measuring the quantities above.

2. PROBLEM AND RELEVANT THEORY

2.1. 21cm Hydrogen Emission

Most hydrogen in the galaxy is found in its ground state. This ground state of hydrogen has two sub-states. Both the proton and electron contained in a hydrogen atom are charged and have spin, and consequently they produce a magnetic moment. The magnetic moments interact, and cause two sub-states: one where two spins are aligned, and one where they are opposite. The state where the two spins are aligned has a higher energy than the anti-aligned state. The difference between the two states corresponds to the emission of 1420.4 MHz radiation. The two sub-states are referred to as the hyperfine split in hydrogen. At $100K$, the temperature of the interstellar medium, $\frac{2}{3}$ of hydrogen atoms are found in the higher energy sub-state. Despite the fact that emission of 1420.4 MHz radiation happens only about every 10^7 years in a single hydrogen atom, a sufficient number of hydrogen atoms transition to produce measurable quantities of this radiation. This is due to the vast number of hydrogen atoms that are present in any galactic line of sight. [2]

By measuring 21cm wavelength radiation, it is possible to obtain interesting features of the galaxy and determine the brightness temperature of the sun.

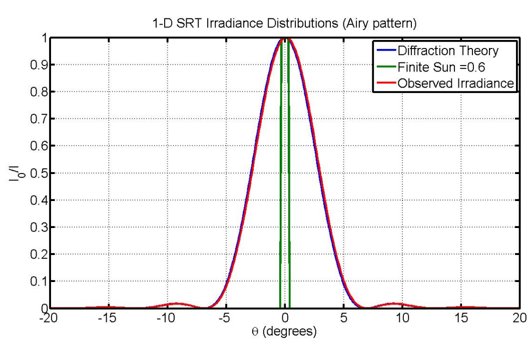

FIG. 1: Theoretical prediction of the diffraction pattern obtained when the radio telescope is pointed to a bright source.

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2.2. Antenna Physics and The Sun

Measurement of the brightness temperature of the sun at 21cm, and optical properties of the telescope are obtained by pointing the antenna at the sun. The antenna can be treated as a circular aperture which will diffract incident light. For a circular aperture, the intensity of light as a function of angle is given by

$$I(\theta) = I_0 \left(\frac{2J_1(kd\sin\theta)}{kd\sin\theta} \right)^2 \quad (1)$$

where k is the wavenumber of incident light, θ is the angle of incidence, d is the diameter of the circular aperture $2J_1(kd\sin\theta)$ is the Bessel function of the first kind of order one.

Equation (1), when plotted with $d = 200cm$, $k = .3\frac{1}{cm}$ is Fig. 1. In this plot, the green top-hat function represents the sun, which covers about .6° in the sky. In our measurements, the sun does not appear as a single line of light, but rather as a beam. In order to account for this, the top-hat of the finite sun is convolved with the predicted diffraction pattern of the telescope. In 1, the blue function is the predicted diffraction function, and the red curve is the green top-hat convolved with the

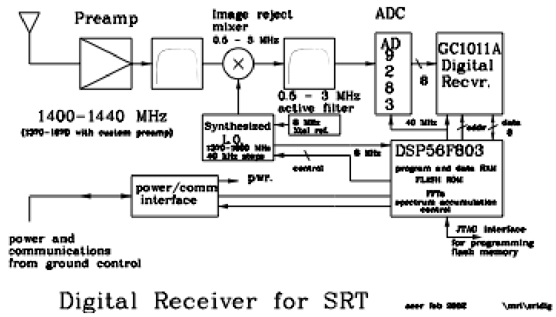

FIG. 2: Block diagram of the experimental telescope apparatus.

predicted diffraction function. Because the red and blue curves are nearly identical, the effect of the convolution will be neglected in further analysis [3].

2.3. Milky Way Galaxy, Visible Matter Rotation Curve

This experiment involves measuring the velocity rotation curve of the galaxy. To understand the acquired data after analysis it is compared to the Keplerian prediction of the rotation curve. The predicted Keplerian curve can be calculated by assuming all of the mass in the galaxy is visible, and that the galaxy is a radially symmetric mass. The latter assumption is valid at large radii since the currently accepted model of the shape of the galaxy is a dense nuclear bulge surrounded by disk with spiral arms. At large radii, the effect of the uniform disk dominates. Assuming uniform circular motion, Newton's law and the law of universal gravitation can be used to calculate the rotation curve:

$$\frac{mv^2}{r} = \frac{GmM}{r^2} \implies v(r) = \sqrt{\frac{GM}{r}} \tag{2}$$

where G is the universal gravitational constant, and M is the visible mass of the galaxy, taken to be 10^{42}kg.

3. EXPERIMENTAL SETUP

3.1. Haystack Observatory

The radio telescope used in this experiment was designed by the Haystack Observatory primarily for educational use. It is a 228cm diameter dish, with a focal length of 85.7cm. The beam width, calculated using the first minimum of the diffraction pattern, is 7°. The telescope, located on the roof of building 26 on the MIT campus is mounted on two motors that can control its azimuth and elevation. The motors are controlled by a micro-controller. The diameter of the aperture of the dish is 228cm, which is one order of magnitude larger than the wavelength radiation (21.5 cm) being observed, therefore the telescope generates a diffraction pattern that modulates the data. This diffraction pattern must be accounted for in the analysis of collected data. [1].

3.2. Signal Chain

The radio antenna picks up a signal at wavelengths near 21cm. As shown in the block diagram in Fig. 2, this signal is reflected and focused to the antenna feed horn. The signal passes through a band-pass filter in order to select a band of the approximate frequency at which data is being taken, a low noise preamplifier, and a mixer. The local oscillator frequency used by the mixer can be modified by the software program, allowing an experimenter to look at varying wavelengths close to 21cm. After passing through the mixer, the signal is sent through a serial cable to a computer running software written in Java.

3.3. SRT Control Software and Data Collection

Remote control of the telescope is managed by a Java applet that interfaces with the micro-controller and telescope receiver. Furthermore, the SRT software allows a user to correct for errors in motor control or communication by setting manual offsets on azimuth and elevation. Additionally, there is built-in functionality to calculate the antenna noise. To calculate background noise the software initiates a noise generator on the dish. The noise source outputs a wide range of frequencies that are distributed like 115K blackbody radiation. The systematic noise is calculated by comparing the received signal when the noise generating source is active to when the noise source is inactive[1].

3.3.1. Galactic Observations

The frequency spectrum for points in the plane of the galaxy were used to measure the rotation curve of the Milky Way. Measurements were taken at 5° increments from 0 to 90°. Each coordinate point was measured for 10 minutes, collecting 156 bins of frequency data every 5 seconds. The plot shown in Fig. 5 is an example of a single 5s collection, with 156 frequency bins from 1419.79 MHz to 1421.01 Mhz, centered around 1420.4 MHz. By taking the average and standard deviation of each bin across the ten minutes of collection, random error and noise is accounted for.

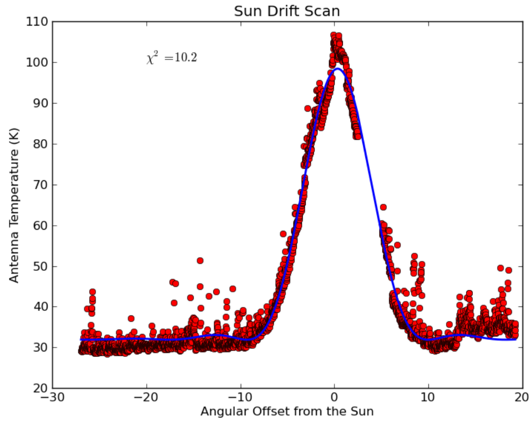

FIG. 3: Sun drift scan obtained October 31, 2010. The gap around $\approx 5°$ degrees of offset was the result of a software issue that caused the telescope to change its location, resulting in a 15 minute data gap in the three hour scan. The telescope was re-centered quickly, and data collection resumed. Despite this data gap, we were able to fit the theoretical functions for the circular aperture.

4. DATA, ANALYSIS, AND ERROR

4.1. Brightness Temperature of the Sun

To calibrate the telescope, approximately 15 npoint scans were executed. The output of the npoint scans is used to calculate the correct offset from the programmed azimuthal and elevation points.

A sun drift scan was taken by pointing the telescope at a position where the sun would pass through an hour and a half later. The scan was taken over the course of three hours. The data is shown in Fig. 3 and was fit to equation (1). Optical properties of the antenna were determined by the best fit curve. By this fit, we were able to determine the effective diameter of the antenna, $d = 255 \pm 3.2cm$, this indicates a discrepancy between our data and the given value of $d = 228cm$, and the half power beam width of the antenna, 7°. This agrees with the half power beam width presented in the Haystack Manual.

Additionally, we used the data in Fig. 3 to determine the brightness temperature of the sun.

$$T_{ant} = \eta T_b^{eff} \tag{3}$$

where $\eta = 0.5$ is the efficiency of the reflector.

Th brightness antenna, calculated from the data show in Fig. 3, needs to be be corrected geometrically. The sun radiates from a surface that is .533°, and the beam width is 7°. This means that the brightness temperature calculated from the antenna temperature is the brightness temperature of the sun plus the area surrounding the sun. The sun radiates from a surface $\pi(L\psi_{sun})^2$ and the view area is $\pi(L\psi_{view})^2$. Therefore the actual brightness temperature of the sun is

$$T_b(\nu_0) = \frac{T_{ant}}{\eta}\left(\frac{\psi_{view}}{\psi_{sun}}\right)^2 \tag{4}$$

where $\psi_{sun} = 32' = 0.533° \pm 0.018°$ is the angular diameter of the sun seen from the earth and $\psi_{view} = 6.41° \pm 0.03°$ is the viewing angle of the telescope.

By using these equations, the brightness temperature of the sun is calculated to be $T_b = (4.5 \pm 0.3)10^4 K$.

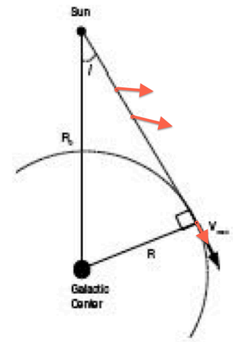

FIG. 4: Schematic for calculation of distance from galactic center. Red arrows indicate direction of velocities.

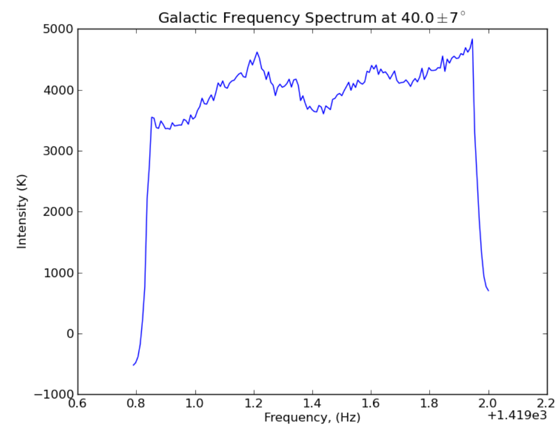

FIG. 5: The raw frequency spectrum of one data point at galactic angle of 40± 7 °.

4.2. Milky Way Galaxy Rotation Curve

Intensity data was collected at frequencies centered at 1420.4 MHz, and any variation from this frequency is a result of doppler shift. By using the doppler shift formula in equation (4), we can calculate the velocity as a function of frequency.

$$v(f) = \frac{c(f_0 - f)}{f_0} - V_{lsr} \tag{5}$$

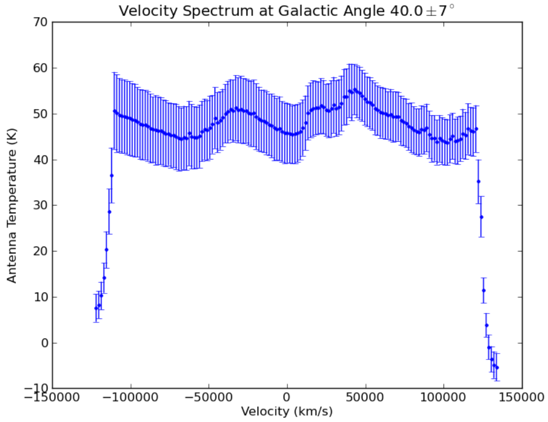

FIG. 6: A plot of the velocity spectrum at the galactic coordinate l=40°. Data was collected at each galactic coordinate for ten minutes, and the values of the data points are the average of the points collected in that time. The error bars are the standard deviation of the same points. The negative values are from the V_{lsr}

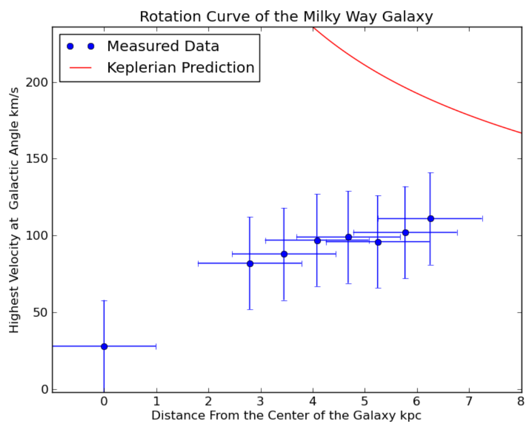

FIG. 7: Rotation Curve of the Milky Way Galaxy, experimental and predicted.

where f_0 is 1420.4 MHz, as explained above, and V_{lsr} is the velocity of the local standard of rest, produced as part of the data collected at each point. The local standard of rest accounts for the movement of the earth around the sun and the sun around the center of the galaxy. The analysis of data is greatly simplified by subtracting V_{lsr} from the stationary-observer doppler shift formula, though it produces some negative velocities as an artifact.

Fig. 6 shows a velocity spectrum with three velocity peaks. Each velocity peak represents an object in the line of sight of the telescope, after the x-axis has been translated from frequency to velocity. There are multiple peaks in each velocity spectrum because there are multiple features in the line of sight of the telescope pointed at any single galactic coordinate. From these plots, we calculate the maximum recessional velocity. In order to plot the rotation curve of the galaxy, there must be specific objects for which both distance and velocity can be calculated. Taking the object with the maximum recessional velocity, the distance of that object from the center of the galaxy can be calculated by trigonometry. This is because objects with maximal recessional velocity are objects that lie on the line tangent to the circle centered at the galactic center, as seen in Fig. 4. The location of the maximum recessional velocity is where the curve can no longer be distinguished from the noise. This determination of maximum velocity was done by hand, and as such, the error bars on the velocities are large, as seen in Fig. 7. Then, having calculated maximum velocity and distance for each of the galactic points, we can compare this to the predicted Keplerian rotation curve, as shown in Fig. 7.

As is obvious from the plot of the rotation curve, the experimental data do not meet the Keplerian prediction. As such, since the main assumption for the Keplerian curve was that the mass of the galaxy is visible, this implies that there must be mass in the galaxy that is not visible. This matter that is not visible is commonly called dark matter, and is the current hypothesis explaining the difference between the two curves.

5. CONCLUSIONS

In this experiment we used a 7.5ft diameter Haystack telescope in order to measure the 21cm line of hyperfine splitting in hydrogen. Using this telescope we were able to obtain a brightness temperature of $T_b = (4.5 \pm 0.3)10^4 K$ for the sun, and a measurement of the diameter of the telescope of $255 \pm 3cm$.

Additionally, we measured the rotation curve of the Milky Way galaxy. Since the rotation curve is not modeled by the Keplerian prediction, it suggests that there is a matter beyond what is visible. The resolution is not sufficient to distinguish between the dark-matter hypothesis, where there is a massive halo of matter that does not interact electromagnetically surrounding the galaxy, and other theories, such a theory involving modified gravity.

[1] *NEROC Haystack Observatory Undergraduate Research Educational Initiative Small Radio Telescope.*

[2] D. Mihalas and J. Binney. *Galactic Rotation and the Spiral Structure of Our Galaxy*, chapter Chapter 8. San Francisco,W.H. Freeman, 1968.

[3] Junior Lab Staff. 21cm astrophysics lab guide, 2010.